Several approaches to solve the rotation illusion with wheel effect

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ABSTRACT

The wheel effect (also called the Wagon-wheel effect) is a well-known rotation illusion in which a rotating wheel, when displayed as individual frames, appears to rotate differently from its true rotation due to temporal aliasing. In this paper, we propose several approaches to solve this problem for synthetic imagery in computer animation. First, we develop an algorithm to compute the frame number at which our visual perception starts to incorrectly interpret the wheel rotation. By making this critical frame number available, we can correct the wheel rotation by manipulating its geometry while viewers are unaware of the change. Our second approach is developed based on the Nyquist sampling theorem. We can increase the sample rate to capture the essential deviation that correctly depicts the wheel rotation to take care of the under-sampling issue. Our third approach is based on the traditional view that texture is often used to aid our motion perception. We further identify certain rules that can be applied to the textures to distinguish the real motion from the illusion. For each approach, we analyze both the advantages and disadvantages and suggest the potential applications.

Keywords: Wheel Effect, Perception, Short-range Apparent Motion, Computer Animation.
Finlay et al. and Purves et al. presented their study of the wheel effect with several key results: the appearance of orthograde and reversed rotation simply alternate, with a strong preference for the direction of real motion; supernumerary elements appear on wagon-wheel illusion; nearest-neighbor principle predicts the direction and speed of movement; currently illuminated spokes and persisting images of spokes induce apparent motion; duration of spoke illumination is a determining factor and so on. In 0, Andrews and Purves concluded temporal parsing of visual information must occur at a minimum every 300ms which is based on the demand of normal saccadic eye movements. Recently, researchers have hotly debated what really causes the illusion in continuous lights in [4], [5], [7], [8], [9] and [10].

1.1 Two categories and their differences

The wheel effect can appear in sequentially displayed images for example, in movies, videos and computer animation. It also occurs in continuous light such as sunlight when viewing, for instance, wheel covers of moving vehicles, propellers of airplanes and so on. From the perception point of view, the phenomena in these two situations have notable differences. In 0 below0, Andrews and Purves address these differences: (1) static patterns are never seen in continuous light; (2) in sequentially displayed images, the stroboscopic effect occurs immediately, whereas, the effect in continuous light does not always appear immediately, but can take time to develop; (3) supernumerary elements (additional spokes) appear by addition in continuous light, whereas they multiply in stroboscopic presentation; and (4) the effect in continuous light is only seen at rates of element presentation between ~2 and 20 Hz; the strobe effect is not limited in this same way.

1.2 Main Causes

The reasons for the illusions in these two settings are also different. Our paper focuses on the wheel effect in sequentially displayed images, so we are interested in only the sources in that category. The temporal aliasing, or under-sampling, is the primary cause of the wagon-wheel phenomenon. When the sampling rate of a camera (frame rate) is too low compared with the spatiotemporal frequency of a rotating object, that is, the sequential images are not enough to capture necessary characteristics of moving object, then the object will appear to be still or to rotate in reverse. The principle of proximity (or the principle of the nearest neighbor) leads to this effect, which determines the direction of the rotation - our visual perception always interpolates the missing intervening frames by the nearest neighbor. Third, as Finlay et al. addressed in [3], the apparent motion is generated between currently illuminated spokes and persisting images of spokes.

1.3 Contribution

To the best of our knowledge, no good solution has been proposed to correct this illusion in computer animation. In this work, we first develop an algorithm to compute the frame number at which our visual perception starts to incorrectly interpret the wheel rotation. Then we propose three approaches to alleviate the wheel effect based on our observations in our experiments. In our first method, starting from the critical frame, we gradually add more spokes to the rotating wheel so that the wheel appears to rotate faster than before. Under this geometry manipulation, we are able to keep the perception of the wheel rotating in the correct direction while the viewer is (usually) unaware of the geometry change of the wheel. Our second approach is developed based on the Nyquist sampling theorem. We can increase the sample rate to capture the essential deviation that correctly depicts the wheel rotation to take care of the under-sampling issue. Our third approach is based on the traditional view that texture is often used to aid our motion perception. We identify certain guidelines that can be applied to the textures to aid a viewer in perceiving the correct motion. Our approaches are efficient in terms of computation time and memory. All of these can be implemented in real time.

1.4 Paper organization

Our experiments and the resulting observations are presented in section 2. In section 3, our main work is addressed as follows: (1) critical frame computation in section 3.1; (2) geometric manipulation in section 3.2; (3) increasing sampling rate in section 3.3; and (3) texture scheme in section 3.4. In section 4, we conclude our work.

2. EXPERIMENTS AND OBSERVATIONS

In order to obtain insights into the phenomenon, we first construct a computer generated animation of a rotating wheel using a three-dimensional geometric model of a wheel and basic principles of physics. Then we investigate the illusion through several experiments.
2.1 Physical and geometric model

The real motion of a wheel is governed by Newton’s second law of motion \( F = ma \). Based on this equation, we have

\[
a = \frac{\partial^2 \omega}{\partial t^2},
\]

where \( a \) is acceleration of the wheel rotation, which is assumed a constant in this model; \( t \) is time, here we refer it to a frame number; \( \omega \), the angle the spoke rotates at a certain frame. We also define \( \theta = \frac{2\pi}{s} \) – the angle between two successive spokes, where \( s \) is the number of spokes of the wheel; \( v \) is the velocity of a wheel. From Newton’s second law, we also know: \( v = at \); then

\[
\omega = \frac{1}{2} at^2.
\]

We construct a geometric model of a wheel using python functions in Maya. The rotation of the wheel is also set up in Maya according to equation (2).

2.2 Experiments

Video 2. Wheel effect 2.

Video 3. Wheel effect 3.

Figure 1. Multiplied spokes appear, i.e. \( 3 \times 6 = 18 \) spokes.

Figure 2. At the critical frame, all angles are roughly same.
We conducted three groups of experiments: general test, speed test and direction test with 12 subjects. All subjects have normal and corrected normal vision. In our algorithm, a user can specify the number of spokes of a wheel and its velocity when creating a rotating wheel. For example, in video 1, the wheel has 8 spokes with \( a = 1.9634/\text{second}^2 \).

2.2.1 General wheel effect

In the general test, we created rotating wheels with 6, 8, 10, 12, and 18 spokes respectively with the same acceleration \( a = 1.9634/\text{second}^2 \). Video 1, Video 2, and Video 3 are a part of this general test. In each animation, we observe: (1) the cycle from the original rotation direction to the still state to the opposite direction is repeated to the end of the animation; (2) we see not only the number of spokes of the wheel but also the multiplication of them over time. Figure 1 shows what we see at frame 22 with the spokes of the previous 20 and 21 frames; (3) when the wheel reaches its critical speed (called the critical frame), we see all angles between two appeared successive spokes are roughly same. Figure 2 illustrates this situation. When a 6-spoke wheel reaches its critical frame 120, we see all angles are about \( \pi/12 \). The perception of the spokes includes all those at frames 117, 118, 119, and 120 frames. This implies that more spokes a wheel has, the faster the wheel will reach its critical frame. In other words, the smaller \( \theta \) is, the smaller value of the critical frame will be.

2.2.2 Speed test

In this experiment, we set all wheels with a constant clock-wise \( v_0 = 2.7317/\text{second} \) and their spoke numbers are 4, 6, 8, and 12 spokes respectively. Video 4 and Video 5 are part of this test. We asked subjects to rank wheel speeds from the lowest to the highest. All subjects report wheel speed is relatively proportional to the number of spokes, that is, a wheel with more spokes appears rotating faster than one with fewer spokes. The reason behind this phenomenon is not clear. We hypothesize that this is because, the perceptual system associates denser geometry with speed. However, this interesting fact leads us to our first approach – geometric manipulation in section 3.2.

2.2.3 Direction test

In this test (See Video 6 and Video 7), we try to figure out whether our perception can use some other information rather than the nearest neighbor to decide the moving direction. When all angles between two appeared successive spokes roughly equal, the principle of proximity is unable to determine the rotation direction. We construct this situation by setting per-frame velocity \( v = \theta/u \), where \( \theta \in [p - 1, p] \). The details of \( p \) are addressed in section 3.1. When the rotation speed equals \( \theta/u \), each spoke simply moves to its appeared neighbor’s position in the next frame. We see only flickering except for the first several frames. The direction test includes two sub-tests. (1) Subjects are blocked from seeing the very beginning of the motion. (2) There is no blocking and the subject starts the video by himself or herself. In the first sub-test (see Figure 4), when the first several frames are blocked, nearly no one can tell the rotating direction from the spokes’ flicking. However, in the second test (see Figure 3), we found most subjects are able to use the initial
movement of spoke to correctly decide the rotating direction. This leads to the conclusion that the initial direction of movement plays a key role in determining the motion direction when the principle of proximity does not function well.

Figure 3. The direction can be determined at the beginning.  
Figure 4. The direction cannot be detected after first 3 frames.

3. THREE APPROACHES TO ALLEVIATE WHEEL EFFECT

Inspired by the above observations, we propose several solutions to the wheel effect. First we design an algorithm to compute the critical frame (in section 3.1). With the critical frame available, we create a geometric manipulation method (in section 3.2) to correct the perceived wheel rotation without viewers’ awareness. Next, we improve the sampling rate to alleviate the wheel effect (in section 3.3). Finally, we employ texture schemes to help our motion perception in wheel illusion.

3.1 Critical frame

The critical frame of the wheel effect is the maximal frame number that our visual system can correctly perceive rotation of a wheel. If this critical frame is available, we know when the perceived motion is wrong. From equation (2),
In addition, the persisting images play an important role in wheel effect. The additional spokes are multiplied by the original number of wheel spokes and a factor, say, \( p \). If \( p \) is given, then \( \Delta \omega = \Delta \omega(t + \Delta t) - \omega(t) = at \cdot \Delta t + 0.5a \cdot \Delta t^2 \) \hspace{1cm} (3)

From our experience, \( p \) is usually 4 with frame rate around 30 when a critical frame is reached. We can use equation (4) to calculate a wheel-effect-free (WEF) acceleration for given \( n \), the number of total frames. Let \( t^* = n \), which means all \( n \) frames are WEF. Then

\[
a_{\text{WEF}} = \frac{\theta p^2}{(n + 0.5) p}
\]

\hspace{1cm} (5)

Video 8 shows a wheel effect with \( a = 1.955 \). Video 9 is WEF with new \( a = 0.588 \) computed in equation (5).

### 3.2 Geometric manipulation

From the results of the speed test, we are able to conclude that the rotation of a wheel with a greater number of spokes appears faster than that of a wheel with the fewer even through both wheels have the same velocity. By using this illusion, we can avoid the perception of the slow speed motion and the opposite rotation after the critical frame in the wheel effect phenomenon. However, how many spokes should be added at a time? How fast should the wheel be taken after adding spokes? How long should it be between two adding spoke manipulations? In sequentially displayed images, supernumerary spokes appear by a multiplication factor. If the number of added spokes equals to the spoke number of the wheel, which is the minimal factor 1, our eyes can detect the manipulation immediately; that is, the resulting rotation is not smooth. Additional spokes in continuous light appear by addition. By gradually adding spokes as if they appear in continuous light, we are able to smooth the resulting rotation. Around the critical frame, the wheel appears stationary except for flickering. The stationary states occur when \( \Delta \omega \mod 2\pi = \theta / u \), or a constant per frame speed with \( v = \theta / u \), where \( u \in [1, p - 1] \). We choose constant \( v_{\text{new}} = \theta_{\text{new}} / (p - 1) \) because it is the smallest angle that the wheel will reach its stationary state. It also provides the best directional cue among all stationary states by the nearest neighbor rule. Even with no clue of the direction at all, our visual perception system can take advantage of the initial movement information to determine the rotation direction, which was demonstrated in the direction test.

Based on the above analysis, we design an algorithm to construct a wheel-effect free (WEF) animation by passing \( a, s, n \) and \( \gamma \) as inputs, where \( a, s, \) and \( \gamma \) are defined above, and \( n \) is the number of total frames (the duration of the animation). The pseudo code of our algorithm is listed in Algorithm 1. The cost of our algorithm is \( O(n) \).
Video 10. Geometric manipulation alleviates the wheel effect.

### Algorithm 1: Geometric manipulation algorithm

**GeometricManipulation**($a, s, n, \gamma, p$)

createWheelEffect($a, s, n, \gamma$); \( \theta = 2\pi/sp; \)

\( i^* = \text{computeCriticalFrame}(a, \theta, \gamma, p); \)

\[ \text{num} = \lceil (n - i^*)/\gamma \rceil ; \]

for \( i \leftarrow 0 \) to \((\text{num} - 1)\)

\( v_i = 2\pi/3s; \) \( \text{start} = iy + i^*; \)

if \((i + 1)\gamma + i^* < n\)

\( \text{end} = (i + 1)\gamma + i^*; \)

else \( \text{end} = n; \)

createWheelWithConstSpeed($v_i, s + 1, \text{start}, \text{end}$.)

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### 3.3 Increasing sampling rate

When the sampling rate of a camera (frame rate) is too low compared with the spatiotemporal frequency of a rotating object, the sequential images cannot capture necessary characteristics of moving object. Figure 5 shows this situation: a 6-spoke wheel with constant velocity \( v = \pi/6 \text{ per frame} \) and frame rate \( \gamma = 25 \), the rotating wheel appears to be still. When applying Nyquist sampling theorem to avoid the temporal aliasing, we must take two elements into account: the number of the identical spokes of the wheel and the supernumerary spokes appearing in the illusion. If a wheel rotates with a constant velocity per frame \( v_f \), then \( v_f < 2\pi/sp \) is a necessary condition to avoid wheel effect. Denote \( k \) to be the factor to fix WE. Then equation (6) guarantees a WEF video with frame rate \( \gamma_{new} \).

\[
v_{f_{new}} < v_{f_{old}}/k \quad \text{and} \quad \gamma_{new} > k \cdot \gamma_{old}, \quad \text{where} \quad k = v_{f_{old}} \cdot sp/2\pi
\]

(6)

Figure 5. The low frame rate cannot capture the characteristics of a moving object. The object seems full-stop. \( v = 30^\circ \text{ per frame.} \)

Figure 6. The frame rate is three times the one in Figure 5. The wheel motion is correctly captured. \( v = 10^\circ \text{ per frame.} \)

Figure 6 shows a WEF wheel follows equation (6). Video 11 demonstrates a wheel effect with \( \gamma = 25 \), whereas Video 12 shows that increasing frame rate from 25 to 75 keeps temporal aliasing from the rotating wheel.

On the other hand, if a wheel has constant acceleration, increasing frame rate is not simply derived from equation (4). The reason is that when frame rate is changed, the value of \( p \) can change as well. Denote \( t \) to be the duration time of animation, then \( t^* \) is the critical time corresponding to \( i^* \). Then equation (4) is equivalent to the following equation:
Let $t_0^*$, $\gamma_0$, and $p_0$ be parameters associated with the known critical frame $i_0^*$. For animation with certain duration time, $t^* = t_0^*$, then

$$t^* = \frac{\theta \gamma - \frac{1}{2\gamma}}{ap}$$

(7)

$p$ is some function of $\gamma$, $p = P(\gamma)$, where the functional form $P(\gamma)$ is not known. However, if this function is known, then $\gamma^*$ or $\gamma_{\text{WEF}}$ can be computed from equation (8). To our best knowledge, how to determine the value of $p$ is not clear. Without knowing this value, increasing frame rate of the wheel with non-zero acceleration seems to be a trial-error process.

Although increasing sampling rate is easy to achieve theoretically in the case in which velocity is constant, this approach is restricted by the available frame-rates of display devices and the capable range of human visual perception.

3.4 Texture scheme

While the characteristics of a texture (i.e., the visual qualities of the surface of the object) can help alleviate the wheel effect to some extent, texture manipulation may be constrained by the intended use of the imagery.

If distinguishing the identical elements (spokes) is the goal, applying various textures to spokes would seem to be an obvious option. Video 13 demonstrates this scheme. Comparing the original wheel effect in Video 8, we can see that the critical frame of the multicolor wheel is prolonged. If the speed and rotating direction are the main concerns, we can make a mark with a texture that is different from the rest of the wheel. We can also take luminance contrast, color oppositeness, etc. into account as well. Video 14 illustrates that the texture with a proper choice of luminance contrast and color prevents wheel effect from occurring in this animation if we focus on the red spoke. This scheme is consistent with equation (4), that is, the value of $\theta$ is enhanced by this scheme.

The gradient color texture is specially good at indicating the rotation direction. For example, the wheel in Video 15 with a gradient color texture shows the rotation direction clearly. Despite the stationary states and reversely rotation from the spokes, the gradient texture grants us a clear hint of the wheel’s rotation direction.
As Bach showed in his website [2], certain visual illusions that induce motion might be useful candidates to alleviate wheel effect. For example, a motion after effect invented by Anstis and Roger, is also showed in Bach’s web site [2]. We modified this illusion as in Video 18. The wheel rotates clockwise while we feel the spokes moves in the opposite direction. It is obvious that the wheel in Video 18 gives better sense of moving direction than those in Video 16 and Video 17. However, how to control the visual illusions and use them precisely in movies, films, and computer animation is still a great challenge for not only film makers and computer scientists but also the scientists in the fields of vision and perception as well.

4. CONCLUSION

In this paper, we present several approaches to solve rotation illusion with wheel effect in computer animation. First, we develop an algorithm to compute the critical frame at which our visual perception starts to incorrectly interpret the wheel rotation. Then we propose three approaches to alleviate the wheel effect based on our observations from experiments. In the method of geometric manipulation, starting from the critical frame, we gradually add more spokes to the rotating wheel so that the wheel appears to rotate faster than before. With this method, we can keep the wheel rotating in the correct direction in our perception while viewers are usually unaware of the geometry change of the wheel. Our second approach is developed based on the Nyquist sampling rule. In the constant velocity case, we can increase the sample rate to capture the essential deviation that correctly depcts the wheel rotation. We also show a potential solution to the case in which acceleration is constant. Our third approach is texture based. We demonstrate some texture schemes to aid our
motion perception. Our approaches are efficient in terms of computation time and memory. They can all be implemented in real time.

REFERENCES